DTEMS PAST PAPER
TECHNICAL

Sir Arthur Lewis Community College

Division of Technical Education and Management Studies

EXAMINATION	:	MAY 2015 - FINAL EXAMINATION
COURSE TITLE	:	CALCULUS II
COURSE CODE	:	MAT 216
TUTORS	:	C. Omerod and N. Serieux
TIME	:	2 HOURS
DATE	:	Friday 8th May 2015
INVIGILATORS	:	P. Jn. Francois, N. Hyacinth; M. Floyd
ROOMS	:	CEHI-1R-02

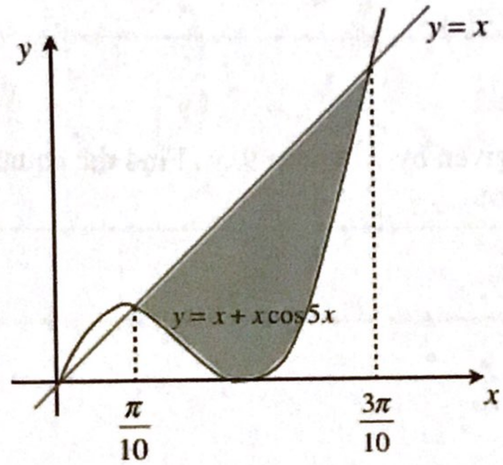
#C69

INSTRUCTIONS:

This exam consists of two sections. Answer all questions in section one and any two in section two.
Answer all questions on the foolscaps provided.
Show all necessary working.
You are permitted to use nonprogrammable calculators.

1. If $-1 \leq x \leq 1$ rewrite $\sin(\arccos x)$ as an algebraic expression in x **[3]**

2.



The diagram above shows the area enclosed by the curve $y = x$ and $y = x + x\cos 5x$.

Find the area of the shaded region. **[4]**

3. The average value of a function f where $a \leq x \leq b$ is defined as

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

In a certain city the temperature (in ${}^0F$) t hours after 9 AM was modeled by the function

$$f(t) = 50 + 14\sin\frac{\pi t}{12}$$

Find the average temperature during the period from 9 AM to 9 PM. **[4]**
(*Give your answer to four decimal places*).

4. We are given that a smooth curve with parametric equations $x = f(t)$, $y = g(t)$, $a \leq t \leq b$ has its length given by

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Find the exact length of the curve $x = 1 + 3t^2, y = 4 + 2t^3, 0 \leq t \leq 1$ **[5]**

5. By use of the substitution $x = 3\sec\theta$, Evaluate the integral $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$ **[6]**

6. a) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$.

b) Find the solution of this equation that satisfies the initial condition when $x = 0$, $y = 2$.

Write equation in terms of y

[6]

7. The equation of a graph is given by $x^3 + y^3 = 9xy$. Find the equation of the tangent to the curve at the point$(2,4)$.

[7]

THE END

List of formulae

Trigonometry

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$1 + \tan^2 x \equiv \sec^2 x$$

$$\cos^2\theta + \sin^2\theta \equiv 1$$

$$\cot^2 x + 1 \equiv cosec^2 x$$

Derivatives

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{(1-x^2)}}$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{(1-x^2)}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

Indefinite Integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \; (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int -\frac{1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$